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A string with a Euclidean worldsheet in $\mathrm{AdS}_5 \times S^5$

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Abstract
This paper is devoted to the study of the classical solutions of a bosonic string with a Euclidean worldsheet in $\mathrm{AdS}_5 \times S^5$. We generalize solutions presented by Kruczenski *et al* to the case where we also include the dynamics of the string on S^5.

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1. Introduction

The AdS/CFT correspondence [1–3][1] shows deep relations between the $N = 4$ super Yang–Mills (SYM) theory and the string theory in $\mathrm{AdS}_5 \times S^5$ where the classical string solutions play an important role [7]; for a review and an extensive list of references, see [8, 9]. The energies of classical strings have been shown to match with the anomalous dimensions of the gauge-invariant operators, while an open string that ends on a curve at the boundary of AdS_5 has been analyzed to study the strong coupling behavior of the Wilson loop in the gauge theory [10–12].

Recently Alday and Maldacena, in a remarkable paper [13][2], computed the planar 4-gluon scattering amplitude at strong coupling in the $N = 4$ SYM theory using AdS/CFT correspondence. The 4-gluon scattering amplitude was evaluated as the string theory computation of the 4-cusp Wilson loop composed of four light-like segments in the T-dual coordinates where a certain open string solution in AdS_5 space is found to minimize the area of the string surface whose boundary conditions are determined by the massless gluon momenta and a dimensional regularization is used to regularize the IR divergence.

As was shown in [32] these results are closely related to the remarkable observations in perturbative (planar $N = 4$) gauge theory: the scaling function $f(\lambda)$ can be either found as a

[1] For review, see [4–6].
[2] For some recent papers, see [15–36].

coefficient in the anomalous dimension of a gauge-invariant large spin twist two operator or as a cusp anomaly of a light-like Wilson line [37, 38]. Then it was shown in [32] that this fact has a nice explanation in dual perturbative $\mathrm{AdS}_5 \times S^5$ where the anomalous dimension of the minimal twist operator is represented either by the energy of a *closed* string with large spin $S \gg 1$ in AdS_5 [7] or it follows the *open* string picture, i.e. from the area of a surface ending on a cusp formed by two light-like Wilson lines on the boundary of AdS_5 [39]. Then it was shown that these two approaches are (under specific scaling limits) closely related when they become equivalent upon certain analytic continuation that is needed to convert the Minkowski worldsheet coordinates in the closed string case into the Euclidean one in the open string Wilson loop case and AdS_5, i.e. conformal $SO(2,4)$ transformation. Then it was shown in [32] that the worldsheet surface studied in [13] can be related (before an IR regularization) to the cusp Wilson loop surface found in [39] using $SO(2,4)$ isometry of AdS_5.

It is remarkable that string with a Euclidean worldsheet that is embedded in spacetime with a Minkowski signature plays such a crucial role in recent developments of string theory. In fact, it is well known that string theories naturally contain in their spectra extended objects with a Euclidean worldsheet signature (S-branes, S-strings [43–45]) even if their precise definition is unclear. On the other hand, we mean that it is certainly important to study properties of these objects and try to identify their possible applications. In fact the goal of this paper is to investigate the dynamics of the bosonic string with the Euclidean worldsheet metric in $\mathrm{AdS}_5 \times S^5$ and try to see how it is possible to extend the classical solution found in [32] to a more general case. Recall that the ansatz given in [32] describes the light-like Wilson line that ends on the boundary of AdS_5. Our goal is to generalize this solution to the case when we allow non-trivial configuration of the string on S^5. We find that in the case of the light-like Wilson line solution the dynamics of the string on AdS_5 decouples from the dynamics on S^5 as a consequence of the fact that the AdS_5 part of the Virasoro constraints vanishes separately. On the other hand, it is important to stress that the solution that describes dynamics of the string on AdS_5 corresponds to *open string* of infinite extent. Then in order to derive the finite value of the string action that is evaluated on the classical solution we have to impose cut-off on the time and space extent of the worldsheet theory. It is clear that the same cut-off has to be performed for the S^5 part of the action as well. Imposing this cut-off we can explicitly evaluate the action on given solutions that presumably give some interesting phenomena in dual CFT.

In order to gain more insight into our solutions we study another class of the Euclidean solution that was given in [14, 32]. This solution arises from Euclidean continuation of the worldsheet time coordinate from the homogeneous solutions [41, 42]. We find that for this solution Virasoro constraints corresponding to dynamics on AdS_5 are non-zero and consequently the solution on AdS_5 is related to the solution on S^5. This result suggests that an equivalence between Euclidean continuation of the homogeneous solutions given in [41, 42] and the light-like Wilson loop solution—where dynamics on AdS_5 decouples from dynamics on S^5—does not generally hold when we include non-trivial dynamics on S^5.

The organization of this paper is as follows. In section 2, we introduce the notation and we solve the equations of motion for the string with a Euclidean worldsheet theory that moves on S^5. We analyze two particular solutions, the first one corresponding to the homogeneous motion and the second one that is analog of the magnon solution given in [40]. In section 3 we outline our results and suggest possible extensions of this work. Finally, in the appendix we review the second class of the Euclidean solution found in [14] and discuss its relation to the solution found in this paper.

2. Euclidean string on AdS$_5$ × S^5

Our goal is to study exact solutions of the closed string theory in AdS$_5$ × S^5 where the fundamental string has worldsheet theory with the Euclidean metric signature while the target spacetime has Minkowski signature. Before we start this analysis we review notations for coordinates in AdS$_5$ [32]. In global coordinates $(\rho, t, \phi, \theta_1, \theta_2)$ the line element of AdS$_5$ takes the form

$$\mathrm{d}s^2 = R^2\big(\mathrm{d}\rho^2 - \cosh^2\rho\,\mathrm{d}t^2 + \sinh^2\rho\big(\mathrm{d}\phi^2 + \cos^2\phi_1\,\mathrm{d}\theta_1^2 + \sin^2\phi\,\mathrm{d}\theta_2^2\big)\big), \tag{2.1}$$

where R is the radius of AdS$_5$ and S^5. It is convenient to introduce the embedding coordinates $X^M, M = (0, \dots, 5)$ on which $SO(4, 2)$ is acting linearly. In these coordinates the line element takes the form

$$\mathrm{d}s^2 = \mathrm{d}X^M \eta_{MN}\,\mathrm{d}X^N, \qquad \eta_{MN} = (-1, 1, 1, 1, 1, -1). \tag{2.2}$$

Note that the global coordinates are related to the embedding coordinates as

$$\begin{aligned} &X^0 + \mathrm{i}X^5 = R\cosh\rho\,\mathrm{e}^{\mathrm{i}t}, \qquad X^1 + \mathrm{i}X^2 = R\sinh\rho\cos\phi\,\mathrm{e}^{\mathrm{i}\theta_1}, \\ &X^3 + \mathrm{i}X^4 = R\sinh\rho\sin\phi\,\mathrm{e}^{\mathrm{i}\theta_2}. \end{aligned} \tag{2.3}$$

We can also introduce Poincaré coordinates where the boundary of AdS$_5$ is at $z = 0$

$$\mathrm{d}s^2 = \frac{1}{z^2}(\mathrm{d}x^m\,\mathrm{d}x_m + \mathrm{d}z^2), \qquad x^m x_m = -x_0^2 + x_i^2, \qquad i = 1, 2, 3, \tag{2.4}$$

where

$$X^0 = \frac{x_0}{z}, \qquad X^i = \frac{x_i}{z}, \qquad X^4 = \frac{1}{2z}(-1 + z^2 + x^m x_m), \qquad X^5 = \frac{1}{2z}(1 + z^2 + x^m x_m). \tag{2.5}$$

Finally, we also use the embedding coordinates $Y^P, P = 0, \dots, 5$ for the embedding coordinates of S^5 with Euclidean metric δ_{PQ}. Then the dynamics of bosonic string on AdS$_5$ × S^5 is governed by the action

$$\begin{aligned} &S = S^{\mathrm{AdS}_5} + S^{S_5}, \\ &S^{\mathrm{AdS}_5} = \frac{1}{4\pi\alpha'}\int \mathrm{d}\sigma\,\mathrm{d}\tau\sqrt{\gamma}(\gamma^{\alpha\beta}\partial_\alpha X^M \partial_\beta X^N \eta_{MN} + \Lambda(X^M \eta_{MN} X^N + R^2)), \\ &S^{S_5} = \frac{1}{4\pi\alpha'}\int \mathrm{d}\tau\,\mathrm{d}\sigma\sqrt{\gamma}(\gamma^{\alpha\beta}\partial_\alpha Y^P \partial_\beta Y^Q \delta_{PQ} + \tilde{\Lambda}(Y^P \delta_{PQ} Y^Q - R^2)), \end{aligned} \tag{2.6}$$

where $\gamma_{\alpha\beta}$ is the worldsheet metric with a Euclidean signature so that in the conformal gauge the line element takes the form $\mathrm{d}s^2 = \mathrm{d}\tau^2 + \mathrm{d}\sigma^2$.

Finally, $\Lambda, \tilde{\Lambda}$ given in (2.6) are Lagrange multipliers that impose conditions $X^M \eta_{MN} X^N = -R^2, Y^P \delta_{PQ} Y^Q = R^2$. Using (2.6) it is easy to determine corresponding equations of motion. The variation of S^{AdS_5} with respect to X^M and with respect to Λ gives

$$\partial_\alpha(\sqrt{\gamma}\gamma^{\alpha\beta}\partial_\beta X^M) - \Lambda X^M = 0, \qquad X^M X_M = -R^2. \tag{2.7}$$

If we multiply the first equation in (2.7) with X_M we obtain

$$\Lambda = \frac{1}{R^2}\partial^a X^M \partial_a X_M, \tag{2.8}$$

where we also used the conformal gauge $\gamma_{\alpha\beta} = \delta_{\alpha\beta}$. In the same way the variation of S^{S_5} with respect to Y^P and $\tilde{\Lambda}$ gives

$$\begin{aligned}
&\partial_\alpha[\delta^{\alpha\beta}\partial_\beta Y^P] - \tilde{\Lambda} Y^P = 0, \\
&Y^P \delta_{PQ} Y^Q = R^2, \qquad \tilde{\Lambda} = -\frac{1}{R^2}\partial_\alpha Y^P \delta_{PQ} \partial_\beta Y^Q \delta^{\alpha\beta}.
\end{aligned} \tag{2.9}$$

Following [32] we consider an ansatz that for $z = 0$ ends on two light-like lines at boundary:

$$z = \sqrt{2}u, \qquad u = \mathrm{e}^{\alpha\tau - \beta\sigma}, \qquad \chi = \alpha\sigma + \beta\tau, \tag{2.10}$$

where α, β are real parameters. If we write the line element in Poincaré coordinates as

$$\mathrm{d}s^2 = \frac{1}{z^2}\left(\mathrm{d}z^2 - \mathrm{d}u^2 + u^2\,\mathrm{d}\chi^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2\right), \qquad x_0 = u\cosh\chi, \qquad x_1 = u\sinh\chi, \tag{2.11}$$

then ansatz (2.10) takes the form

$$x_0 = \mathrm{e}^{\alpha\tau - \beta\sigma}\cosh(\alpha\sigma + \beta\tau), \qquad x_1 = \mathrm{e}^{\alpha\tau - \beta\sigma}\sinh(\alpha\sigma + \beta\tau), \qquad x_2 = x_3 = 0. \tag{2.12}$$

Finally, in the embedding coordinates ansatz (2.10) takes the form

$$\begin{aligned}
&X^0 = \frac{R}{\sqrt{2}}\cosh(\alpha\sigma + \beta\tau), \qquad X^5 = \frac{R}{\sqrt{2}}\cosh(\alpha\tau - \beta\sigma), \\
&X^1 = \frac{R}{\sqrt{2}}\sinh(\alpha\sigma + \beta\tau), \qquad X^4 = \frac{R}{\sqrt{2}}\sinh(\alpha\tau - \beta\sigma), \qquad X^2 = X^3 = 0.
\end{aligned} \tag{2.13}$$

By analogy with (2.13) we propose the following ansatz for the motion of string on S^5:

$$\begin{aligned}
&Y^1 = \frac{R}{\sqrt{2}}\cos(\gamma\tau + \delta\sigma), \qquad Y^2 = \frac{R}{\sqrt{2}}\sin(\gamma\tau + \delta\sigma), \\
&Y^3 = \frac{R}{\sqrt{2}}\cos(\gamma\sigma - \delta\tau), \qquad Y^4 = \frac{R}{\sqrt{2}}\sin(\gamma\sigma - \delta\tau),
\end{aligned} \tag{2.14}$$

where γ, δ are constants. Since we consider open string with infinite extent then the solution above describes string that wraps infinitely many times compact space S^5.

Now it is easy to see that for ansatz (2.13) Λ takes the form

$$\Lambda = (\alpha^2 + \beta^2) \tag{2.15}$$

and consequently ansatz (2.13) solves the equation of motion (2.7). In the same way (2.14) gives

$$\tilde{\Lambda} = -(\gamma^2 + \delta^2) \tag{2.16}$$

and again it is easy to see that the equation of motion (2.9) are satisfied as well.

As the next step we impose Virasoro constraints. Explicitly, the variation of action (2.6) with respect to worldsheet metric $\gamma^{\alpha\beta}$ implies the constraints

$$T_{\alpha\beta} = \frac{2\pi}{\sqrt{\gamma}}\frac{\delta S}{\delta\gamma^{\alpha\beta}} = T_{\alpha\beta}^{\mathrm{AdS}_5} + T_{\alpha\beta}^{S_5} = 0, \tag{2.17}$$

where

$$\begin{aligned}
&T_{\tau\tau}^{\mathrm{AdS}_5} = -T_{\sigma\sigma}^{\mathrm{AdS}_5} = \frac{1}{2\alpha'}(\partial_\tau X^M \partial_\tau X_M - \partial_\sigma X^M \partial_\sigma X_M), \\
&T_{\tau\sigma}^{\mathrm{AdS}_5} = \frac{1}{\alpha'}\partial_\tau X^M \partial_\sigma X_M, \\
&T_{\tau\tau}^{S_5} = -T_{\sigma\sigma}^{S_5} = \frac{1}{2\alpha'}(\partial_\tau Y^P \partial_\tau Y_P - \partial_\sigma Y^P \partial_\sigma Y_P), \\
&T_{\tau\sigma}^{S_5} = \frac{1}{\alpha'}\partial_\tau Y^P \partial_\sigma Y_P,
\end{aligned} \tag{2.18}$$

where we considered the metric in the form $\gamma_{\alpha\beta} = \mathrm{diag}(1, 1)$ and in the final step we used the equation of motion for Λ and $\tilde{\Lambda}$. Now for ansatz (2.13) we obtain

$$T^{\mathrm{AdS}_5}_{\tau\tau} = 0, \qquad T^{\mathrm{AdS}_5}_{\tau\sigma} = 0. \tag{2.19}$$

On the other hand, the S^5 part of the Virasoro constraints imply

$$T^{S_5}_{\tau\tau} = 0, \qquad T^{S_5}_{\tau\sigma} = 0 \tag{2.20}$$

and we see that these constraints vanish identically as well. However the analysis performed above suggests that the dynamics on AdS_5 decouples from the dynamics on S^5 for ansatz (2.13). On the other hand, when we evaluate action on given solutions we have to take integration cut-off into account in a sense that we presume that the string has a finite extent. In fact note that the AdS_5 part of the action is equal to

$$S^{\mathrm{AdS}_5} = \frac{\sqrt{\lambda}}{4\pi}(\alpha^2 + \beta^2)\int \mathrm{d}\tau\,\mathrm{d}\sigma, \tag{2.21}$$

where $\sqrt{\lambda} = \frac{R^2}{\alpha'}$. We see that in order to find a finite value of action (2.21) we have to presume that the time and spatial coordinates that parameterize a string worldsheet are bounded. Then we mean that it is natural that the same integration bounds have to be imposed both in AdS_5 and in S^5 parts of the action.

As in [32] we introduce the following cut-off prescription:

$$\ln l < \alpha\tau - \beta\sigma < \ln L, \qquad -\frac{\Gamma}{2} < \alpha\sigma + \beta\tau < \frac{\Gamma}{2}. \tag{2.22}$$

As the next step we introduce the coordinates

$$\begin{aligned} m &= \alpha\tau - \beta\sigma, & n &= \alpha\sigma + \beta\tau, \\ \tau &= \frac{\alpha m + \beta n}{\alpha^2 + \beta^2}, & \sigma &= \frac{n\alpha - \beta m}{\alpha^2 + \beta^2}, \end{aligned} \tag{2.23}$$

where the Jacobian of the transformation from (τ, σ) to (m, n) is equal to $J = \frac{1}{\alpha^2+\beta^2}$. Then we can easily evaluate (2.21) and we obtain

$$\begin{aligned} S^{\mathrm{AdS}_5} &= \frac{\sqrt{\lambda}}{4\pi}(\alpha^2 + \beta^2)\int \mathrm{d}\tau\,\mathrm{d}\sigma \\ &= \frac{\sqrt{\lambda}}{4\pi}(\alpha^2 + \beta^2)\int_{\ln l}^{\ln L} \mathrm{d}m \int_{-\frac{\Gamma}{2}}^{\frac{\Gamma}{2}} \mathrm{d}n\, J \\ &= \frac{\sqrt{\lambda}}{4\pi}\Gamma \ln\frac{L}{l}. \end{aligned} \tag{2.24}$$

We see that the value of the action does not depend on α, β. On the other hand, when we evaluate S^{S_5} for ansatz (2.14) we obtain

$$S^{S_5} = \frac{\sqrt{\lambda}}{4\pi}\int \mathrm{d}\tau\,\mathrm{d}\sigma\,(\gamma^2 + \delta^2) = \frac{\sqrt{\lambda}}{4\pi}\frac{\gamma^2 + \delta^2}{\alpha^2 + \beta^2}\Gamma \ln\frac{L}{l}. \tag{2.25}$$

However it would be more natural to express given action in terms of conserved charges. In fact, it is easy to see that the action S^{S_5} is manifestly invariant under rotation

$$Y'^M = \Omega^M_N Y^N \approx Y^M + \omega^M_N Y^N, \qquad \omega^M_N = -\omega^N_M \ll 1 \tag{2.26}$$

that implies an existence of following conserved charges:

$$J^{MN} = \frac{1}{4\pi\alpha'}\int \mathrm{d}\sigma\,(Y^M\partial_\tau Y^N - Y^N\partial_\tau Y^M). \tag{2.27}$$

Let us now define

$$J_1 = J^{12}, \qquad J_2 = J^{34}, \qquad J_3 = J^{56}. \tag{2.28}$$

Then for (2.14) we obtain

$$J_1 = \frac{\sqrt{\lambda}}{8\pi}\gamma \int d\sigma, \qquad J_2 = -\frac{\sqrt{\lambda}}{8\pi}\delta \int d\sigma. \tag{2.29}$$

It turns out, however, that for the integration domain defined in (2.22) these charges explicitly depend on time. For that reason we restrict ourselves to the case when $\beta = 0$. Then $-\frac{\Gamma}{2\alpha} < \sigma < \frac{\Gamma}{2\alpha}$ and we obtain

$$\begin{aligned} J_1 &= \frac{\sqrt{\lambda}}{8\pi}\gamma \int_{-\frac{\Gamma}{2\alpha}}^{\frac{\Gamma}{2\alpha}} d\sigma = \frac{\sqrt{\lambda}}{8\pi}\frac{\gamma}{\alpha}\Gamma, \\ J_2 &= -\frac{\sqrt{\lambda}}{8\pi}\delta \int_{-\frac{\Gamma}{2\alpha}}^{\frac{\Gamma}{2\alpha}} d\sigma = -\frac{\sqrt{\lambda}}{8\pi}\frac{\delta}{\alpha}\Gamma, \end{aligned} \tag{2.30}$$

and hence we can write

$$S^{S_5} = 16\frac{\pi}{\sqrt{\lambda}}\left(J_1^2 + J_2^2\right)\frac{1}{\Gamma}\ln\frac{L}{l}. \tag{2.31}$$

In summary, we obtain that the action evaluated on the solution is equal to

$$S = \frac{\sqrt{\lambda}}{4\pi}\ln\frac{L}{l}\Gamma + 16\frac{\pi}{\sqrt{\lambda}}\frac{1}{\Gamma}\ln\frac{L}{l}\left(J_1^2 + J_2^2\right). \tag{2.32}$$

To conclude, we found generalization of the solution [32] where we included non-trivial dynamics on S^5. We have also shown that this solution is valid for any values of parameters α, β and that Virasoro constraints do not imply any relation between the motion on AdS_5 and S^5. On the other hand, we have argued that when we wanted to evaluate worldsheet action on these solutions we had to impose the same integration cut-off in both parts of the action.

Let us now consider ansatz that is an analog of the magnon-like solution [40]. To do this we restrict ourselves to the motion of the string on S^2. Then it is convenient to use the parameterization

$$Y^1 = R\sin\theta\cos\phi, \qquad Y^2 = R\sin\theta\sin\phi, \qquad Y^3 = R\cos\theta, \tag{2.33}$$

so that the line element on two sphere S^2 takes the form

$$ds^2 = R^2(d\theta^2 + \sin^2\theta \, d\phi^2). \tag{2.34}$$

Then the action that determines motion on S^2 takes the form

$$S^{S_2} = \frac{R^2}{4\pi\alpha'}\int d\tau\, d\sigma[\delta^{\alpha\beta}\partial_\alpha\theta\partial_\beta\theta + \sin^2\theta\delta^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi]. \tag{2.35}$$

Further we presume that the motion on AdS_5 is determined by ansatz (2.13) while for the motion on S^2 we propose an ansatz

$$\theta = \theta(y), \qquad \phi = \omega\tau + \tilde{\phi}(y), \tag{2.36}$$

where

$$y = \gamma\tau + \delta\sigma. \tag{2.37}$$

Using the simple form of action (2.35) it is easy to determine corresponding equations of motion for ϕ

$$[\sin^2\theta((\gamma^2 + \delta^2)\tilde{\phi}' + \omega\gamma)]' = 0, \tag{2.38}$$

while the equation of motion for θ takes the form

$$(\gamma^2+\delta^2)\theta''-\sin\theta\cos\theta((\omega+\gamma\tilde{\phi}')^2+\delta^2\tilde{\phi}'^2)=0, \tag{2.39}$$

where $(\cdots)'\equiv\frac{\mathrm{d}(\cdots)}{\mathrm{d}y}$. Note that (2.38) implies

$$\tilde{\phi}'=\frac{1}{\gamma^2+\delta^2}\left[\frac{B}{R^2\sin^2\theta}-\omega\gamma\right], \tag{2.40}$$

where B is a constant. Further, using the fact that $T^{\mathrm{AdS}_5}_{\alpha\beta}$ vanishes separately we use the Virasoro constraints $T^{S_2}_{\tau\tau}=0$ in order to determine differential equation for θ

$$\begin{aligned}\theta'^2&=-\frac{\sin^2\theta}{\gamma^2-\delta^2}[(\omega+\gamma\tilde{\phi}')^2-\delta^2\tilde{\phi}'^2]\\&=-\frac{1}{(\gamma^2+\delta^2)^2}\left[\frac{B^2}{R^4\sin^2\theta}-\omega^2\delta^2\sin^2\theta+\frac{4\gamma B\omega\delta^2}{R^2(\gamma^2-\delta^2)}\right]\end{aligned} \tag{2.41}$$

using (2.40). On the other hand, when we consider the second Virasoro constraint $T^{S_2}_{\tau\sigma}=0$ we obtain

$$T^{S_2}_{\tau\sigma}=\theta'^2\gamma\delta+\sin^2\theta(\omega\delta\tilde{\phi}'+\gamma\delta\tilde{\phi}'^2)=0 \tag{2.42}$$

that together with the constraint $T^{S_2}_{\tau\tau}=0$ implies

$$\tilde{\phi}'=-\frac{\omega\gamma}{\gamma^2+\delta^2} \tag{2.43}$$

and consequently

$$\phi=\frac{\omega\delta}{\gamma^2+\delta^2}(\delta\tau-\gamma\sigma). \tag{2.44}$$

Further, when we compare (2.43) with (2.40) we obtain that $B=0$ and consequently (2.41) implies the following differential equation:

$$\theta'=\frac{\omega\delta}{\gamma^2+\delta^2}\sin\theta \tag{2.45}$$

that has the solution

$$\cos\theta=-\frac{\sinh\left(\frac{\omega\delta}{\gamma^2+\delta^2}(\gamma\tau+\delta\sigma)\right)}{\cosh\left(\frac{\omega\delta}{\gamma^2+\delta^2}(\gamma\tau+\delta\sigma)\right)}. \tag{2.46}$$

Let us now evaluate the S_2 part of the action for ansatz (2.36)

$$\begin{aligned}S^{S_2}&=\frac{\sqrt{\lambda}}{4\pi}\int\mathrm{d}\sigma\,\mathrm{d}\tau[\theta'^2(\gamma^2+\delta^2)+\sin^2\theta[(\omega+\gamma\tilde{\phi}')^2+\delta^2\tilde{\phi}'^2]]\\&=\frac{\sqrt{\lambda}}{2\pi}\int\mathrm{d}\tau\,\mathrm{d}\sigma\frac{\omega^2\delta^2}{\gamma^2+\delta^2}\sin^2\theta.\end{aligned} \tag{2.47}$$

In order to evaluate action (2.47) appropriately we have to impose the integration cut-off that arise from the analysis of the dynamics of AdS_5 string and we obtain

$$S^{S_2}=\frac{\sqrt{\lambda}}{2\pi}\frac{\omega^2\delta^2}{\gamma^2+\delta^2}\int_{\frac{1}{\alpha\ln l}}^{\frac{1}{\alpha}\ln L}\mathrm{d}\tau\int_{-\frac{\Gamma}{2\alpha}}^{\frac{\Gamma}{2\alpha}}\mathrm{d}\sigma\frac{1}{\cosh^2\left(\frac{\omega\delta}{\gamma^2+\delta^2}(\gamma\tau+\delta\sigma)\right)}=\frac{\sqrt{\lambda}}{\pi\alpha}\ln\frac{L}{l}. \tag{2.48}$$

Let us calculate the charge related to the isometry along the ϕ direction

$$\begin{aligned} J_\phi &= \frac{\sqrt{\lambda}}{2\pi}\int_{-\frac{\Gamma}{2\alpha}}^{\frac{\Gamma}{2\alpha}} \mathrm{d}\sigma \sin^2\theta(\omega+\gamma\phi') \\ &= \frac{\sqrt{\lambda}}{2\pi}\frac{\delta^2\omega}{\gamma^2+\delta^2}\int_{-\frac{\Gamma}{2\alpha}}^{\frac{\Gamma}{2\alpha}} \mathrm{d}\sigma \frac{1}{\cosh^2\left(\frac{\omega\delta}{\gamma^2+\delta^2}(\gamma\tau+\delta\sigma)\right)} = \frac{\sqrt{\lambda}}{\pi} \end{aligned} \tag{2.49}$$

and hence we obtain the result that

$$S^{S_2} = J_\phi \frac{1}{\alpha}\ln\frac{L}{l}. \tag{2.50}$$

Interestingly, due to the profile of the classical solution the action does not depend on the spatial cut-off Γ. It would certainly be interesting to find the dual CFT interpretation of such a configuration.

It is important to stress that for $\alpha = \beta = 1$ ansatz (2.13) is related to ansatz (A.5) where $\tilde{\alpha} = \tilde{\beta} = 1$. In other words, the scaling limit of the spinning closed string solution is equivalent, upon an analytic continuation to the Euclidean worldsheet combined with a discrete $SO(2, 4)$ rotation in AdS_5, to the global AdS_5 version of the null cusp solution given in (2.13) (for $\alpha = \beta = \tilde{\alpha} = \tilde{\beta} = 1$). However, as was shown above in the case when we include non-trivial dynamics on S^5 this is not generally true since now $\tilde{\alpha}, \tilde{\beta}$ are functions of the conserved charges related to the dynamics on S^5, while in the case of the null cusp solution (2.13) the dynamics on AdS_5 decouples from the dynamics on S^5 at least on the classical level.

3. Conclusion

In this section we give a short summary of results derived in this paper. Our goal was to study some solutions of *open string* with a Euclidean worldsheet that propagates in $AdS_5 \times S^5$. We studied the solution presented in [32] and we found that this solution is valid for any real parameters α, β. We have also shown that Virasoro constraints for this solution vanish. This has an important consequence when we included non-trivial dynamics on S^5 that now naively decouples from the dynamics on AdS_5. Since we imposed the cut-off on the string in the sense of the finite extent of the worldsheet we showed that the actions evaluated on the classical solutions contain the same cut-offs and hence give the same significant contributions to scattering amplitudes in dual quantum field theory. It would certainly be very interesting to clarify the relation of the solutions found there to scattering phenomena or expectation values of Wilson loops in dual QFT living on the boundary of AdS_5.

Then we also considered a second form of the solution that was presented in [14]. We have shown that this solution has a non-zero contribution from the AdS_5 part and hence the motion on S^5 does not decouple from the motion on AdS_5. We calculated the values of worldsheet actions on these solutions. Again, it would be desirable to find interpretations of these solutions in dual QFT theory where the dynamics on S^5 corresponds to non-trivial expectation values of scalar fields in dual $N = 4$ SYM theory.

To conclude we mean that the dynamics of Euclidean string in $AdS_5 \times S^5$ has many interesting properties that deserve further study.

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Appendix. Review of the second type of Euclidean solution [14]

Now we review and also generalize the second type of the Euclidean worldsheet solution [14]. We start with the equations of motion for bosonic string with Minkowski metric

$$\begin{aligned}&\partial_\alpha[\eta^{\alpha\beta}\partial_\beta X^M]-\frac{1}{R^2}(\eta^{\alpha\beta}\partial_\alpha X^N\partial_\beta X_N)X^M=0,\\&\partial_\alpha[\eta^{\alpha\beta}\partial_\beta Y^P]+\frac{1}{R^2}(\eta^{\alpha\beta}\partial_\alpha Y^Q\partial_\beta Y_Q)Y^P=0,\end{aligned}\tag{A.1}$$

where $\eta=\mathrm{diag}(-1,1)$. Let us then consider the following ansatz[3]:

$$\begin{aligned}&X^0=R\cosh\tilde{\alpha}\sigma\cos\tilde{\beta}\tau', &&X^1=R\sinh\tilde{\alpha}\sigma\cos\tilde{\beta}\tau',\\&X^5=R\cosh\tilde{\alpha}\sigma\sin\tilde{\beta}\tau', &&X^2=R\sinh\tilde{\alpha}\sigma\sin\tilde{\beta}\tau',\end{aligned}\tag{A.2}$$

where τ' is now time coordinate on Minkowski worldsheet. We again find that the equations of motion are satisfied for any $\tilde{\alpha},\tilde{\beta}$. As the next step we perform an analytic continuation $\tau'=-\mathrm{i}\tau$ and hence

$$\begin{aligned}&X^0=R\cosh\tilde{\alpha}\sigma\cosh\tilde{\beta}\tau, &&X^1=R\sinh\tilde{\alpha}\sigma\cosh\tilde{\beta}\tau,\\&X^5=\mathrm{i}R\cosh\tilde{\alpha}\sigma\sinh\tilde{\beta}\tau, &&X^2=\mathrm{i}R\sinh\tilde{\alpha}\sigma\sinh\tilde{\beta}\tau'.\end{aligned}\tag{A.3}$$

If we now write $X^2=\mathrm{i}X'^5$, $X^5=\mathrm{i}X'^2$ we obtain

$$\begin{aligned}&X'^0=R\cosh\tilde{\alpha}\sigma\cosh\tilde{\beta}\tau, &&X'^1=R\sinh\tilde{\alpha}\sigma\cosh\tilde{\beta}\tau,\\&X'^2=R\cosh\tilde{\alpha}\sigma\sinh\tilde{\beta}\tau, &&X'^5=R\sinh\tilde{\alpha}\sigma\sinh\tilde{\beta}\tau.\end{aligned}\tag{A.4}$$

Finally we perform rotation in (0, 5) and (1, 2) planes and we obtain

$$\begin{aligned}X^0&=\frac{X'^0+X'^5}{\sqrt{2}}=\frac{R}{\sqrt{2}}\cosh(\tilde{\alpha}\sigma+\tilde{\beta}\tau),\\X^5&=\frac{X'^0-X'^5}{\sqrt{2}}=\frac{R}{\sqrt{2}}\cosh(\tilde{\alpha}\sigma-\tilde{\beta}\tau),\\X^1&=\frac{X'^1+X'^2}{\sqrt{2}}=\frac{R}{\sqrt{2}}\sinh(\tilde{\alpha}\sigma+\tilde{\beta}\tau),\\X^2&=\frac{X'^1-X'^2}{\sqrt{2}}=\frac{R}{2}\sinh(\tilde{\alpha}\sigma-\tilde{\beta}\tau).\end{aligned}\tag{A.5}$$

Let us now check properties of ansatz (A.5). Firstly, it is easy to see that Λ is equal to

$$\Lambda=(\tilde{\alpha}^2+\tilde{\beta}^2)\tag{A.6}$$

and hence the equations of motions are satisfied. On the other hand, the Virasoro constraints $T_{\tau\tau}^{\mathrm{AdS}_5}=-T_{\sigma\sigma}^{\mathrm{AdS}_5}=0$ is equal to

$$T_{\tau\tau}^{\mathrm{AdS}_5}=\frac{\sqrt{\lambda}}{4}(\tilde{\beta}^2-\tilde{\alpha}^2)\tag{A.7}$$

and we see that the AdS_5 part of the stress–energy tensor does not vanish. Then let us again consider the motion on S^5 that is parameterized with ansatz (2.14). In this case, Virasoro constraints $T_{\tau\tau}=T_{\tau\tau}^{\mathrm{AdS}_5}+T_{\tau\tau}^{S_5}=0$ implies

$$\tilde{\beta}^2-\tilde{\alpha}^2+\gamma^2-\delta^2=0.\tag{A.8}$$

[3] Solution (A.2) belongs to the class of homogeneous string solutions as the rigid circular string found in [41, 42].

Consequently the action evaluated on ansatz (2.14) and (A.5) is equal to

$$S = \frac{\sqrt{\lambda}}{4\pi}[\tilde{\beta}^2 + \tilde{\alpha}^2 + \gamma^2 + \delta^2] \int d\tau\, d\sigma$$
$$= \frac{\sqrt{\lambda}}{2\pi\tilde{\beta}^2}[\tilde{\beta}^2 + \gamma^2]\Gamma \ln \frac{L}{l}, \tag{A.9}$$

where now we chose the integration cut-off $-\frac{\Gamma}{2\tilde{\beta}} < \sigma < \frac{\Gamma}{2\tilde{\beta}}$, $\frac{1}{\tilde{\beta}}\ln l < \tau < \frac{1}{\tilde{\beta}}\ln L$. Then we introduce charges [9]

$$S_0 = \frac{1}{2\pi\alpha'}\int d\sigma\,(X^5\partial_\tau X^0 - X^0\partial_\tau X^5), \qquad S_1 = \frac{1}{2\pi\alpha'}\int d\sigma\,(X^1\partial_\tau X^2 - X^2\partial_\tau X^1) \tag{A.10}$$

that for (A.5) are equal to

$$S_0 = \tilde{\beta}\frac{\sqrt{\lambda}}{4\pi}\int d\sigma\, \sinh 2\tilde{\alpha}\sigma$$
$$= \frac{\sqrt{\lambda}}{4\pi}\frac{\tilde{\beta}}{\tilde{\alpha}}\cosh\frac{\tilde{\alpha}}{\tilde{\beta}}\Gamma \approx \frac{\sqrt{\lambda}}{4\pi}\frac{\tilde{\beta}}{\tilde{\alpha}}e^{\frac{\tilde{\alpha}}{\tilde{\beta}}\Gamma}, \qquad S_1 = -S_0. \tag{A.11}$$

In the same way we find that J_1, J_2 defined in (2.27) and (2.28) are equal to

$$J_1 = \frac{\sqrt{\lambda}}{8\pi}\frac{\gamma}{\tilde{\beta}}\Gamma, \qquad J_2 = -\frac{\sqrt{\lambda}}{8\pi}\frac{\delta}{\tilde{\beta}}\Gamma. \tag{A.12}$$

Then we can write the action in an alternative form

$$S = \frac{8\pi}{\sqrt{\lambda}}S_0^2\, e^{-2\frac{\tilde{\alpha}}{\tilde{\beta}}\Gamma}\Gamma \ln\frac{L}{l} + \frac{32\pi}{\sqrt{\lambda}}\frac{J_1^2}{\Gamma}\ln\frac{L}{l}. \tag{A.13}$$

It is important to stress that now $\tilde{\alpha}$ and $\tilde{\beta}$ are not arbitrary but are determined by charges J_1, J_2 through relation (A.8). In particular, the condition $\tilde{\alpha} = \tilde{\beta} = 1$ can be imposed in the case when $\gamma^2 - \delta^2 = 0$ (equivalently when $J_1^2 = J_2^2$). We will discuss consequence of this result below.

Let us again consider the magnon-like solution where now we have to take into account the non-zero contribution from the AdS_5 part of the Virasoro constraint that we denote as $T_{\mathrm{AdS}_5} \equiv \frac{1}{4\alpha'}\kappa^2$. Then the vanishing of the total $T_{\tau\tau} = T_{\tau\tau}^{\mathrm{AdS}_5} + T_{\tau\tau}^{S^2} = 0$ implies

$$\theta'^2 = -\frac{\kappa^2}{R^2(\gamma^2 - \delta^2)} - \frac{\sin^2\theta}{\gamma^2 - \delta^2}[(\omega + \gamma\tilde{\phi}')^2 - \delta^2\tilde{\phi}'^2]$$
$$= -\frac{1}{(\gamma^2 + \delta^2)^2}\left[\frac{B^2}{R^4\sin^2\theta} - \omega^2\delta^2\sin^2\theta + \frac{\kappa^2}{R^2}(\gamma^2 - \delta^2)\right]. \tag{A.14}$$

On the other hand, the second Virasoro constraint $T_{\tau\sigma} = 0$ implies

$$T_{\tau\sigma} = \theta'^2\gamma\delta + \sin^2\theta(\omega\delta\tilde{\phi}' + \gamma\delta\tilde{\phi}'^2) = 0 \tag{A.15}$$

that together with the constraint $T_{\tau\tau} = 0$ gives

$$\kappa^2\gamma + B\omega = 0 \tag{A.16}$$

using also the fact that the equation of motion for ϕ takes the form

$$\tilde{\phi}' = \frac{1}{\gamma^2 + \delta^2}\left[\frac{B}{R^2\sin^2\theta} - \omega\gamma\right]. \tag{A.17}$$

Finally, we obtain

$$\theta'^2 = \frac{\omega^2\delta^2}{(\gamma^2+\delta^2)^2\sin^2\theta}[(\sin^2\theta - M)(\sin^2\theta - N)],$$
$$M = -\frac{\kappa^2}{\omega^2 R^2}, \qquad N = \frac{\kappa^2\gamma^2}{\delta^2 R^2\omega^2}. \tag{A.18}$$

We see that in order to have a real solution we should perform an analytic continuation

$$\omega = \mathrm{i}\tilde{\omega}, \qquad \gamma = \mathrm{i}\tilde{\gamma}. \tag{A.19}$$

Then we finally obtain

$$\theta' = \frac{\tilde{\omega}\delta}{(\delta^2 - \tilde{\gamma}^2)\sin\theta}\sqrt{(\sin^2\theta - \sin^2\theta_1)(\sin^2\theta_2 - \sin^2\theta)}, \tag{A.20}$$

where

$$\sin^2\theta_1 = \frac{\kappa^2}{\tilde{\omega}^2 R^2}, \qquad \sin^2\theta_2 = \frac{\kappa^2\tilde{\gamma}^2}{R^2\delta^2\tilde{\omega}^2}. \tag{A.21}$$

Following [47] we can now distinguish two limiting configurations: either the giant magnon solution where $\sin^2\theta_1 = 1$ or the spike solution where $\sin^2\theta_2 = 1$. Let us firstly consider the giant magnon solution. The action evaluated on this solution takes the form

$$S = \frac{\sqrt{\lambda}}{4\pi}\int \mathrm{d}\tau\,\mathrm{d}\sigma\left[(\delta^2 - \tilde{\gamma}^2)\theta'^2 + \sin^2\theta\frac{\tilde{\omega}^2\delta^2}{\delta^2 - \tilde{\gamma}^2} + \frac{B^2}{(\delta^2 - \tilde{\gamma}^2)R^4\sin^2\theta}\right]$$
$$= \frac{\tilde{\omega}^2\delta^2\sqrt{\lambda}}{4\pi(\delta^2 - \tilde{\gamma}^2)}\int \mathrm{d}\tau\,\mathrm{d}\sigma\frac{\sin^2\theta(1+\sin^2\theta_2)}{\sin^2\theta} = \frac{\sqrt{\lambda}}{2\pi}\frac{\tilde{\omega}^2(\delta^2+\tilde{\gamma}^2)}{(\delta^2 - \tilde{\gamma}^2)}\frac{\Gamma}{\tilde{\beta}}\ln\frac{L}{l}. \tag{A.22}$$

Interestingly, for the spike solution $\left(\sin^2\theta_2 = \frac{\kappa^2\tilde{\gamma}^2}{R^2\delta^2\tilde{\omega}^2} = 1\right)$ we find

$$S = \frac{\sqrt{\lambda}}{2\pi}\frac{\tilde{\omega}^2\delta^2(\delta^2+\tilde{\gamma}^2)}{\tilde{\gamma}^2(\delta^2 - \tilde{\gamma}^2)}\frac{\Gamma}{\tilde{\beta}}\ln\frac{L}{l}, \tag{A.23}$$

where now $\tilde{\omega}$ is related to $\kappa^2 = R^2(\tilde{\beta}^2 - \tilde{\alpha}^2)$ through relation (A.16).

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